8   *The Griffith concept*

Fig. 1.5. Energetics of Griffith crack in uniform tension. Data for glass from Griffith's paper: $\gamma = 1.75$ Jm^{-2}, $E = 6.2 \times 10^{10}$ Nm^{-2}, $\sigma_L = 2.63 \times 10^6$ Nm^{-2} (selected to give equilibrium at $c = 10$ mm).

The Griffith equilibrium condition (1.6) may now be applied to (1.10); this gives as a critical condition for fracture

$$\sigma_L = (2E\gamma/\pi c)^{1/2} \qquad (1.11)$$

for constant load, plane stress conditions. Reference to (1.8) indicates that the corresponding result for plane strain may be obtained simply by replacing E by $E/(1 - \nu^2)$. As we may see from fig. 1.5, or from the negative value of d^2U/dc^2, the system energy is a maximum at equilibrium, so the configuration is *unstable*. In particular, if the applied stress exceeds the critical level of (1.11), the crack is free to propagate spontaneously without limit.

Turning to experimentation, Griffith prepared glass fracture specimens from thin round tubes and spherical bulbs. Cracks of length 4–23 mm were introduced with a glass cutter and the specimens annealed prior to testing. The hollow tubes and bulbs were then burst by pumping in a fluid, and the critical stresses determined from the internal fluid pressure. As predicted, only the stress component normal to the crack plane was found to be important; the application of end loads to tubes containing longitudinal cracks had no detectable effect on the critical conditions.

1.4   *Obreimoff's experiment*   9

The results were found to obey the relationship

$$\sigma_L c^{1/2} = 2.63 \times 10^5 \text{ Nm}^{-3/2}$$

with a maximum scatter of about five per cent, thus verifying the essential *form* of (1.11).

If we now take this result, along with Griffith's measured value of Young's modulus, $E = 6.2 \times 10^{10}$ Nm^{-2}, and insert into (1.11), we obtain $\gamma = 1.75$ Jm^{-2} as an estimate of the surface energy of glass. Griffith attempted to substantiate his model by obtaining a second, independent estimate of γ. He measured the surface tension within the temperature range 1020–1383 K, where the glass flows easily, and extrapolated linearly back to room temperature to find $\gamma = 0.54$ Nm^{-1} (Jm^{-2}). Considering that even present-day techniques are incapable of measuring surface energies of solids to much better than a factor of two, the agreement within an order of magnitude between the above estimates of γ represents a triumph for the Griffith theory.

1.4   Obreimoff's experiment

The case of a plane crack in uniform tension represents just one application of the energy-balance equation (1.6). In order to emphasise the generality of the Griffith concept we digress briefly to discuss an important experiment carried out by Obreimoff (1930) on the cleavage of mica. This second example provides an interesting contrast to the one treated by Griffith, in that the equilibrium configuration proves to be *stable*.

The basic arrangement used by Obreimoff is shown in fig. 1.6. A glass wedge of thickness h is inserted beneath a thin flake of mica attached to a parent block, and is made to drive a crack along the cleavage plane.
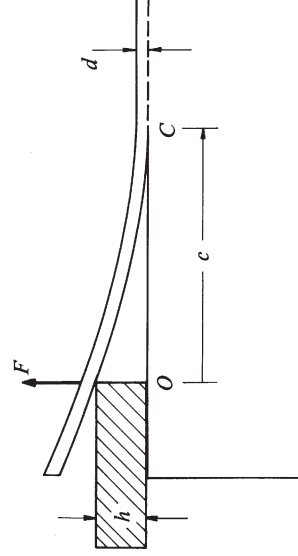

Fig. 1.6. Obreimoff's experiment on mica. Wedge inserted to peel off cleavage flake.

In this case we may determine the energy of the crack system by treating the cleavage lamina as a freely-loaded cantilever, of thickness d and width unity, built-in at the crack front distant c from the point of application of the wedge. We note that in allowing the crack to form under constant wedging conditions the bending force F suffers no displacement, so the net work done on the system is simply

$$W_L = 0. \qquad (1.12)$$

At the same time we have, from simple beam theory, the elastic strain energy in the cantilever arm,

$$U_E = Ed^3h^2/8c^3. \qquad (1.13)$$

Again, the surface energy is

$$U_S = 2c\gamma. \qquad (1.14)$$

The total system energy (1.5) now follows, and application of the Griffith condition (1.6) leads finally to the equilibrium crack length

$$c = (3Ed^3h^2/16\gamma)^{1/4}. \qquad (1.15)$$

The mechanical energy, surface energy and total energy are plotted in fig. 1.7. It is evident from the minimum in U that (1.15) corresponds to a stable configuration. In this instance the fracture is 'controlled'; the crack advances into the material no faster than the rate of entry of the wedge.

Equation (1.15) indicates that, as in Griffith's uniform tension example, a knowledge of equilibrium crack geometry uniquely determines the surface energy. Obreimoff proceeded thus to evaluate the surface energy of mica under different test conditions, and found a dramatic increase from $\gamma = 0.38 \, \mathrm{Jm^{-2}}$ at normal atmosphere ($10^5 \, \mathrm{N\,m^{-2}}$ pressure) to $\gamma = 5.0 \, \mathrm{Jm^{-2}}$ in vacuum ($10^{-4} \, \mathrm{N\,m^{-2}}$). The test *environment* was clearly an important factor to be considered in evaluating material strength. Moreover, Obreimoff noticed that on insertion of the glass wedge the crack did not grow immediately to its equilibrium length: in air equilibrium was reached within seconds, whereas in vacuum the crack continued to creep for several days. Thus the time element was another complicating factor to be considered. These observations provided the first indication of the important role of *chemical kinetics* in fracture processes.

Obreimoff also observed phenomena which raised the question of *reversibility* in crack growth. For a start, propagation of the crack was often acc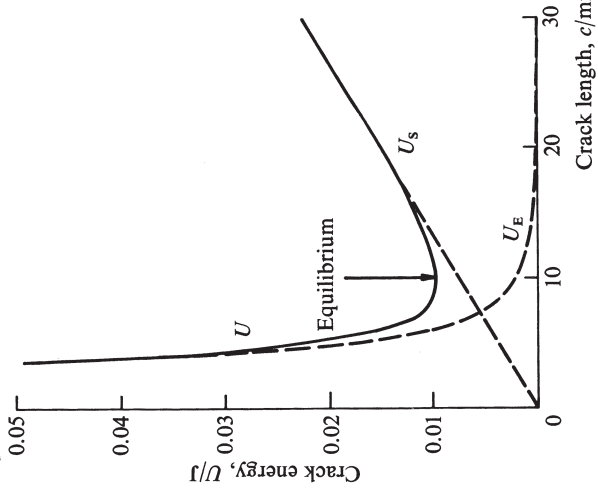ompanied by a visible electrostatic discharge ('triboluminescence'). Secondly, on partial withdrawal of the glass wedge in a vacuum test the crack was observed to retreat and apparently 'heal', but re-insertion of the wedge revealed a perceptible reduction in cleavage strength. These results imply the existence in the energy balance of dissipative terms.

Fig. 1.7. Energetics of Obreimoff crack. Data for mica from Obreimoff's paper: $\gamma = 0.38 \, \mathrm{Jm^{-2}}$ (air), $E = 20 \times 10^{10} \, \mathrm{Nm^{-2}}$, $h = 0.48 \, \mathrm{mm}$, $d = 75 \, \mu\mathrm{m}$ (selected to give equilibrium at $c = 10 \, \mathrm{mm}$).

1.5   Molecular theory of strength

Although Griffith formulated his basic criterion for fracture in terms of macroscopic thermodynamical quantities, he was well aware that a complete description required an evaluation of events at the molecular level. He argued that the maximum stress at the tip of an equilibrium crack must correspond to the *theoretical strength* of the solid, that is, the largest possible stress level that the molecular structure can sustain by virtue of intrinsic bond strength. Griffith accordingly estimated the theoretical strength of his glass from the stress-concentration formula (1.4), inserting $\rho \approx 5 \times 10^{-10} \, \mathrm{m}$ (molecular dimensions), as a reasonable tip radius for a crack growing by a sequential bond-rupture mechanism, together with his measured value of $\sigma_L c^{1/2}$ (§1.3). The value obtained, $\sigma_{yy} \approx 2.3 \times 10^{10} \, \mathrm{Nm^{-2}}$, is an appreciable fraction of the Young's

modulus for glass, representing a strain of some 0.3–0.4. Griffith appreci-ated that Hooke's law could hardly be assumed to hold at such stress levels, for the force–separation relationship for interatomic bonds surely becomes *nonlinear* just prior to rupture. Nor could (1.4), based on the continuum concept of matter, be relied upon to give accurate results on the molecular scale. Making due allowance for these factors, Griffith considered $E/10$ to be an estimate of a reasonable order of magnitude for the theoretical strength.

By way of confirmation of his estimate, Griffith consulted the literature for values of the 'intrinsic pressure' of solids (as determined, for instance, from the heat of vaporisation or equation of state): since both the theoretical strength and intrinsic pressure essentially measure the molecular cohesion, their magnitudes should be comparable, at least for nearly isotropic solids. This was found to be the case. Griffith thus inferred that the theoretical strength should be a material constant, closely related to the energy of the cohesive bonds, with a value close to one tenth of the elastic modulus for all solids.

Thus with both ρ and σ_{yy} effectively predetermined by the molecular structure of the solid, the critical applied tension in the Inglis equation (1.4) becomes dependent on the crack *size*. The last obstacle to a basic fracture criterion (§1.1) is thereby removed.

1.6   Griffith flaws

The argument in the previous section gave an indication of the strength that could be achieved by an ideal solid, an ultimate target in the fabrica-tion of strong solids. Griffith was intrigued by the fact that the strengths of 'real materials' fell well short of this level, typically by two orders of magnitude, despite great care in maintaining specimen perfection on an optical scale. A further discrepancy was also evident. If a solid were to fail at its theoretical strength the applied stress would reach a maximum at rupture, implying a zero elastic modulus at this point: at such a rupture point a sudden release of stored elastic strain energy, equivalent approximately to the heat of vaporisation, would be expected to manifest itself as an explosive separation of the constituent atoms. Again, real materials behaved differently, parting instead with relatively little kinetic energy on a more or less well-defined plane.

Griffith could only conclude that the typical brittle solid must contain a profusion of submicroscopic *flaws*, microcracks or other centres of heterogeneity too small to be detected by ordinary means. The 'effective length', c_f, of these so-called 'Griffith flaws' was calculated by inserting the tensile strength of the strongest as-received glass specimen tested

(§1.3), $\sigma_L = 1.7 \times 10^8$ Nm^{-2}, along with the previously measured values of E and γ, into the critical condition (1.11): this gave $c_f \approx 2$ μm†. We may deduce from (1.2) that a molecularly sharp microcrack of this length has a wall separation $2b \approx 0.05$ μm, which is about a tenth of the wavelength of visible light and therefore barely on the limit of optical detectability. The stress concentration factor (1.4) is of order 100 in this instance, emphasising the potential weakening power of even the most minute flaw.

To test his flaw hypothesis Griffith ran a series of experiments on the strength of glass fibres. The fibres were drawn from the same glass as used in the previous tests (§1.3), and broken either in tension or in bending under monotonically increasing dead weight. Well-prepared, pristine fibres showed unusually high strengths, shattering in the explosive manner expected of ideal, flawless solids. However, on exposure to laboratory atmosphere all fibres declined steadily in strength, reaching after a few hours a 'steady state' value more typical of ordinary glass specimens. Griffith next tested a large number of such 'aged' fibres with diameters ranging from 1 mm down to 3 μm, and found an apparent *size effect*; the thinner specimens showed a tendency to greater strength. Arguing that a single chain of molecules must possess the theoretical strength (being incapable of sustaining a flaw), he extra-polated his data to molecular dimensions, and once again arrived at a value close to one tenth of the elastic modulus. Thus in the one series of tests Griffith had demonstrated convincingly not only that sources of weakness exist in the average specimen, but also that these could be avoided if sufficient care and skill be exercised in preparation. The manufacture of the composite material 'fibre glass', in which strong, pristine glass fibres are incorporated into a tough, protective resin mat-rix, is a modern exploitation of this principle.

It remained only for Griffith to speculate on the *genesis* of these flaws. He actually rejected the possibility that the flaws might be real microcracks, since the observed decrease in fibre strength with time would require the system energy to increase spontaneously by the amount of surface energy of the crack faces. He also rejected the possibility that the flaws might generate spontaneously by stress-assisted thermal fluctuations, regarding as highly improbable the synchronised rupture of a large number (say 10^8) of neighbouring bonds, except perhaps at temperatures close to the melting point. Griffith considered that the most likely explanation lay in the highly localised rearrangement of

† Although the nature of the flaws may in reality be far removed from that of a true microcrack, it is convenient to interpret the stress-concentrating power in terms of the 'effective length' of an 'equivalent microcrack'.

molecules within the glass network, with transformations from the metastable, amorphous state into a higher density, crystalline phase (devitrification). He envisaged sheet-like units with an associated internal field capable of nucleating full-scale fractures. As we shall see in the following chapter, Griffith's speculations on the origin and nature of flaws have largely been superseded: the idea of the flaw as a source of weakness in a solid has, nevertheless, played a vital part in the historical development of present-day fracture theory.

1.7 Further considerations

With his energy-balance concept (pertaining to crack *propagation*) and flaw hypothesis (pertaining to crack *initiation*), Griffith had laid a solid foundation for a general theory of fracture. In a second paper in 1924 he developed his ideas still further, giving explicit consideration to the effect of applied *stress state* on the critical fracture conditions, and discoursing on the factors which determine *brittleness*. With regard to the stress state, Griffith extended his analysis of §1.3 to the case of a biaxial applied stress field, such that the crack plane is subjected to both normal stress (tension *or* compression) and shear stress (parallel to crack growth direction) components. Referring once more to the Inglis stress analysis of an elliptical hole, he argued that the location of the local tension at the crack tip, hence the direction of crack extension, will rotate away from the major axis of the ellipse as the shear component is made larger. From this approach conclusions concerning the crack path and critical applied loading could be drawn. A somewhat surprising result which emerges from the analysis is that the crack tip may develop high local tension even when both principal stresses of the applied field are compressive, provided they are unequal.

As to the question of brittleness, Griffith could but touch on the complications which were apparent in the fracture of many different material types, including some of great engineering importance. In many structural steels, for instance, the incidence of plastic flow prior to or during rupture was known to have a profound effect on the strength, but there seemed no way of reconciling this behaviour with the energy-balance model. It will be recalled that Griffith had based his model on the idea of a reversible thermodynamical system, in which the creation of new fracture surface by the conservative rupture of cohesive bonds provided the sole mode of absorbing mechanical energy. This may be taken as a means of defining an 'ideally brittle' solid. In 'real materials', however, irreversible processes inevitably accompany crack growth, and a substantially greater amount of mechanical energy may be consumed

in the process of separating the material. Thus it was recognised that different materials might exhibit different 'degrees of brittleness': a theoretical understanding of this factor remained an important problem for the future.

What follows in the subsequent chapters is the logical extension of the theory of brittle fracture from the fundamental concepts expounded by Griffith. The chapter headings and subheadings will themselves serve to indicate the broad manner in which the subject has since developed.